HIBA Math Olympiad (HMO)

Sample Paper Grade 9

Pattern and Marking Scheme				
Grade	**Topic / Section**	**NO. of Questions**	**Marks Per Questions**	**Total Marks**
Grade 9	Practical Mathematics	40	1	40
	Achiever's Section	10	2	20
Grade Total		**50**		**60**

The total duration of the exam is 60 minutes. Grade 9 (Age 14–15)

Syllabus

Section 1: Number Systems, Polynomials, Coordinate Geometry, Linear Equations in Two Variables, Introduction to Euclid's Geometry, Lines and Angles, Triangles, Quadrilaterals, Areas of Parallelograms and Triangles, Circles, Constructions, Heron's Formula, Surface Areas and Volumes, Statistics, Probability.

Achievers Section: Higher Order Thinking Questions - Syllabus as per Section 1

Each Question is 1 Mark

1. ABCD is a rectangle formed by the points A(0, 0), B(0, 6), C(8, 6) and D(8, 0). P, Q, R and S are midpoints of AB, BC, CD and DA respectively. The quadrilateral PQRS is a:

   a. Square b. Rectangle c. Rhombus d. None of these

2. The points A(1, 2), B(2, 4), C(6, 6) and D(4, 5) are such that:

   a. A, B, C and D are collinear
   b. ABCD is a parallelogram
   c. D lies inside triangle ABC
   d. D lies on the boundary of triangle ABC

3. A rectangular box has dimensions p, q and r units with $p < q < r$. If one dimension is increased by 1 unit, the increase in volume is:

   a. Greatest when p is increased.
   b. Greatest when q is increased.
   c. Greatest when r is increased.
   d. The same regardless which one is increased.

4. Polynomials $ax^2 + 4x^2 - 5$ and $2x^3 - 7x + a$ when divided by $(x - 3)$ leave remainders R1 and R2 respectively. If $3R1 - R2 = 0$, then a = :

   a. $-\frac{10}{89}$ b. $\frac{10}{23}$ c. $\frac{11}{89}$ d. $-\frac{11}{23}$

5. If $x + y = 15$, then $x + y + z = 15 + z$. Which Euclid axiom illustrates this?

   a. First axiom b. Second axiom c. Third axiom d. Fourth axiom

6. Which of the following are correct?

   I. If two sides of a triangle are unequal, the larger side has the greater opposite angle.

   II. The sum of any two sides of a triangle is greater than the third side.

III. From an external point to a line, the perpendicular segment is the shortest.

IV. If all three sides of a triangle are equal, it is called a scalene triangle.

a. I and III b. I, II and III c. I, III and IV d. Only III

7. ABCD is a parallelogram. If its diagonals are equal in length, what is measure of angle ABC?

a. 70° b. 80° c. 90° d. 100°

8. The point (3, y) divides the line joining $(-6, 2)$ and $(6, 2)$ in what ratio and what is y?

a. 3:2, y = 2 b. 2:3, y = 2 c. 3:2, y = 3 d. 2:3, y = 3

9. An urn contains 5 blue and P green balls. If probability of drawing a green ball is twice that of drawing a blue, then P =:

a. 5 b. 15 c. 20 d. 10

10. Which relationship is correct?

a. $P(E) + P(\bar{E}) = 1$ b. $P(\bar{E}) - P(E) = 1$

c. $P(E) = 1 + P(\bar{E})$ d. None of these

11. $\frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\} = ?$

a. $a^3 + b^3 + c^3 + 3abc$ b. $a^3 + b^3 + c^3 - 3abc$

c. $a^3 + b^3 + c^3 + 3abc(a+b+c)$ d. $3abc$

12. If $x = a(b-c)$, $y = b(c-a)$, $z = c(a-b)$ then $(\frac{x}{a})^3 + (\frac{y}{b})^3 + (\frac{z}{c})^3 = ?$

a. $\frac{3xyz}{abc}$ b. $\frac{xyz}{abc}$ c. $3xyzabc$ d. 3

13. A bag contains 4 red, 5 yellow and 3 green balls. If two balls are picked at random, probability that both are red or both are green is:

a. $\frac{3}{8}$ b. $\frac{5}{16}$ c. $\frac{1}{8}$ d. $\frac{1}{4}$

14. A box has 30 eggs; 10 are rotten. Two eggs selected at random. Probability exactly one is rotten:

a. $\frac{11}{29}$ b. $\frac{17}{29}$ c. $\frac{13}{29}$ d. $\frac{32}{87}$

15. Two years ago $A:B = 6:11$. A's age three years ago was 14 years less than B's age six years ago. Find B's present age.

a. 36 b. 30 c. 34 d. 32

16. If $\frac{(4+3\sqrt{7})}{(4-3\sqrt{7})} = a + b\sqrt{7}$, then $(a, b) =$:

a. $(\frac{83}{41}, -\frac{24}{41})$ b. $(-\frac{83}{41}, \frac{24}{41})$

c. $(\frac{83}{41}, \frac{24}{41})$ d. $(-\frac{83}{41}, -\frac{24}{41})$

17. $\sqrt{7+4\sqrt{3}}$ equals:

a. $\sqrt{3}+2$ b. $\sqrt{3}-\sqrt{2}$ c. $\sqrt{2}-\sqrt{3}$ d. $\sqrt{3}+\sqrt{2}$.

18. Simplify $(a+b)^3+(a-b)^3+6a(a^2-b^2)$:

a. $8a^2$ b. $8a^2b$ c. $8a^3b$ d. $8a^3$

19. If $x+y+z=1$, $xy+yz+zx=-1$, $xyz=-1$, then $x^3+y^3+z^3=?$

a. -1 b. 1 c. 2 d. -2

20. ABCD is a rhombus with $\angle ABC = 52°$. Then $\angle ACD =$:

a. 90° b. 60° c. 52° d. 64°

21. A tells truth 62% of time, B 72%. Probability they say same about one event:

a. 0.56 b. 0.54 c. 0.38 d. 0.94

22. If x^3+6x^2+11k leaves remainder $-2x$ when divided by x^2+3, $k =$:

a. -3 b. -1 c. 1 d. 2

23. G is centroid of triangle ABC; area of $ABC = 84\ cm^2$. Area of $\triangle BDG =$:

a. 12 cm² b. 14 cm² c. 21 cm² d. 7 cm²

24. From four corners of a square side 6 cm, four quarter-circle pieces radius 3 cm are cut. Area remaining =:

a. $18-3\pi$ b. $36-9\pi$ c. $36-6\pi$ d. $9-3\pi$

25. Measurements: one side $+4\%$ and other side -3%. Percentage error in area $\approx$:

a. 1.1% b. 0.9% c. 1.0% d. 1.2%

26. Difference between squares of two consecutive even integers is divisible by:

a. 3 b. 4 c. 6 d. 7

27. Area of rectangle is $506\ m^2$. Length is 20% more than breadth. Breadth =:

a. 20 m b. 23 m c. 26 m d. 21 m

28. Numerator is 5 less than denominator. If numerator -2 and denominator $+1$ makes denominator $7\times$ numerator, find fraction.

a. $\frac{3}{8}$ b. $\frac{4}{9}$ c. $\frac{2}{9}$ d. $\frac{3}{7}$

29. If $x = 4 + 3\sqrt{2}$, find $x^2 + \frac{1}{x^2}$:

a. 65 b. 68 c. 70 d. 66

30. Factorize $11a^2 - 14\sqrt{2a} + 4$:

a. $(11a - 2\sqrt{2})(a - \sqrt{2})$ b. $(11a - \sqrt{2})(a - 2\sqrt{2})$

c. $(11a - 2)(a - 2\sqrt{2})$ d. $(11a - 2\sqrt{2})(a - 2)$

31. Region $x + y > 9$ and $x + y < 13$ in first quadrant represents a:

a. triangle b. rectangle c. trapezium d. rhombus

32. A fraction becomes 1 if $+1$ numerator and -1 denominator; becomes $\frac{1}{3}$ if $+1$ to denominator. Sum numerator+denominator =:

a. 7 b. 8 c. 9 d. 11

33. Speed of Karina is 6 km/h more than Anna. Anna reaches 3 hours earlier than Karina. If Anna and Karina are 30 km and 60 km from offices respectively, speed of Karina =:

a. 12 km/h b. 6 km/h c. 10 km/h d. 9 km/h

34. Lines $4x + 3y - 8 = 0$ and $8x + ky - 10 = 0$ are such that any line perpendicular to the first is also perpendicular to the second. k =:

a. -6 b. -8 c. 6 d. 8

35. Circumcenter of triangle $O(0,0), A(8,0), B(0,8)$ is:

a. (4,4) b. (2,2) c. (1,1) d. (0,0)

36. Students ratio $C1: C2 = 3:5$. After shifting 15 from $C1$ to $C2$ ratio becomes $4:9$. How many must be shifted from $C2$ to $C1$ to get $11:13$?

a. 10 b. 15 c. 20 d. 8

37. Train A crosses a 150 m platform in 75 s. Train B speed = 2×A. A's length is 80% of B. B crosses 300 m platform in x s. Find x.

a. 30 b. 35 c. 40 d. 45

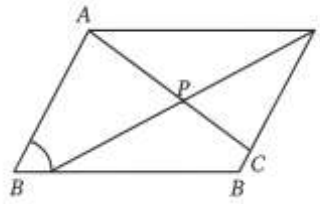
38. $\frac{3}{5} + \frac{(-4)}{7} = ?$ The result being $\frac{-1}{35}$ shows:

a. Rational numbers closed under addition b. Not closed under addition

c. Closed under multiplication

d. Addition not commutative

39. In parallelogram $ABCD$, AP and BP bisect $\angle A$ and $\angle B$ meeting at P. Then $2\angle APB =$?



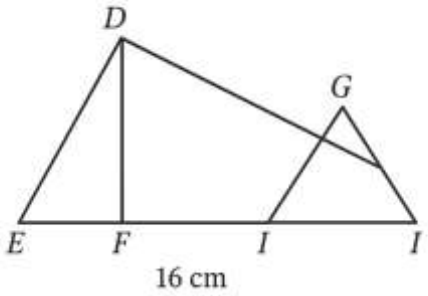
a. $\angle C + \angle D$

b. $\angle A + \angle C$

c. $\angle B + \angle D$

d. $2 \times \angle C$

40. Two isosceles right triangles DEF and HGI on same base $DH, DH \,||\, FI. If\ DE = GH = 6\ cm$ and $DH = 16\ cm$, area of quadrilateral $FEGI =$:



a. 72 cm²

b. 24 cm²

c. 48 cm²

d. 96 cm²

Each Question is 2 Mark

41. If $(x^3 + ax^2 + bx + 6)$ when divided by $(x^2 + x - 2)$ leaves no remainder (i.e. the quotient is a polynomial of degree 1), find a and b.

a) $a = -1,\ b = -4$ b) $a = -2,\ b = -5$

42. c) $a = 3,\ b = -4$ d) $a = 3,\ b = 4$

43. In right triangle ABC with $AB = 3, BC = 4, AC = 5$. Let D be the midpoint of AC. A square is constructed with side BD. What is the area of the square?

a) $4.0\ cm^2$ b) $6.25\ cm^2$ c) $5.5\ cm^2$ d) None of these

43. A right triangular prism has base sides $6,\ 8,\ 10$ and height $15\ cm$. It is transformed into a prism whose base sides are $9,\ 12,\ 15$ (same shape scaled). Find the height of the new prism so that volumes are equal, and find the change in total surface area (new − original).

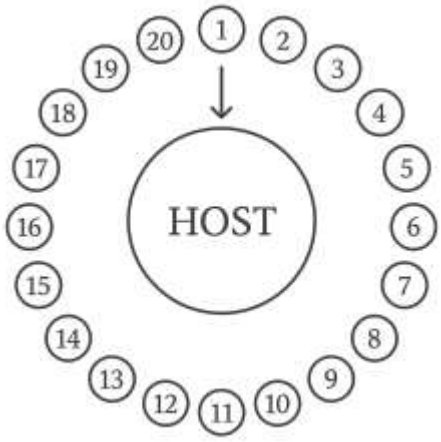
a) $10\ cm,\ +120\ cm^2$

b) $8\ cm,\ +132\ cm^2$

c) $\frac{20}{3}\ cm,\ -60\ cm^2$

d) $8\ cm,\ -120\ cm^2$

44. Twenty people (including the host) are to be seated around a circular table. Two particular guests must sit on either side of the host. How many seating arrangements satisfy this condition (count rotations as identical)?



a) $2 \times 18!$ b) $2 \times 17!$ c) $17!$ d) $18!$

45. A person invested \$5,000 at simple interest 6% per annum for 2 years. At the end of 2 years he took the entire amount (principal+interest) and invested it at compound interest 10% per annum for 2 more years. What is the total amount at the end of 4 years?

a) \$6,560 b) \$6,686 c) \$6,600 d) \$6,776

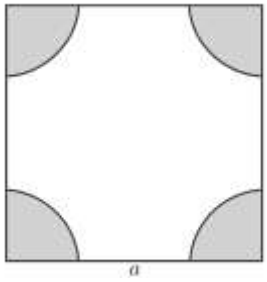
46. The value of a gemstone varies as the cube of its weight. The gem breaks into three pieces in ratio $1:2:3$. If the original (unbroken) gem was worth **\$96,336**, what is the loss in value due to breakage?

a) \$80,280 b) \$16,056 c) \$40,140 d) \$8,028

47. In triangle PQR, a line LM is drawn parallel to QR so that trapezium $LMRQ$ has area twice the area of triangle PLM. If $\frac{PL}{PQ} = k$, find k.

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$

48. A square of side a has at each corner a circular sector constructed so that the corner side a is a chord subtending 120° at the sector's centre. (So each corner contributes the sector minus the triangular part.) Express the total shaded+square area as a simple formula; which of the following is correct?



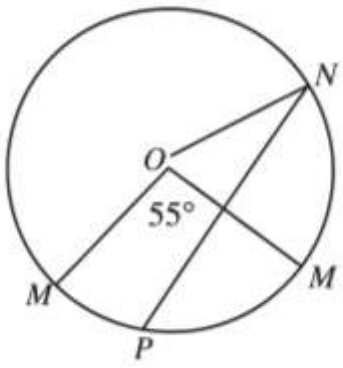
a) $a^2 + 4(\frac{a^2\pi}{9} - \frac{a^2}{\sqrt{23}})$

b) $a^2 + 4(\frac{a^2\pi}{9} - \frac{a}{12}\sqrt{3})$

c) $9a^2 - 4\pi + \sqrt{33}\,a^2$

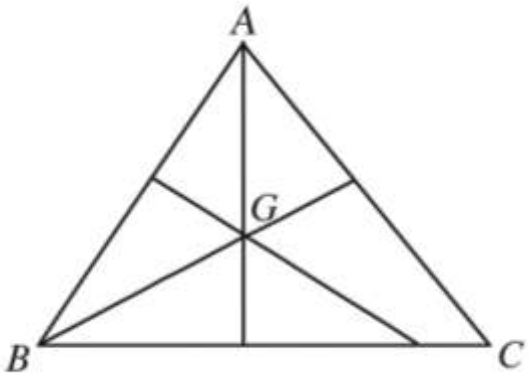
d) None of these

49. A circle has center O. Points M, N, P lie on the circumference. If the inscribed angle $\angle MPN = 55^\circ$ what is the value of $\angle MON + \angle OMN + \frac{1}{2}\angle MNO$?



a) 145° b) 162.5° c) 158.5° d) 180°

50. In triangle ABC the medians intersect at centroid G. If area $(\triangle ABC) = 90\ cm^2$, what is the area of the triangle formed by joining the midpoints of the sides of ABC?



a) $22.5\ cm^2$ b) $30\ cm^2$ c) $45\ cm^2$ d) $15\ cm^2$

Answer Key

1.	a	2.	c	3.	c	4.	b	5.	a	6.	b	7.	c
8.	b	9.	d	10.	a	11.	b	12.	a	13.	c	14.	b
15.	a	16.	c	17.	d	18.	d	19.	d	20.	d	21.	a
22.	b	23.	b	24.	b	25.	a	26.	b	27.	b	28.	a
29.	d	30.	a	31.	c	32.	b	33.	c	34.	a	35.	a
36.	c	37.	c	38.	a	39.	a	40.	c	41.	b	42.	b
43.	c	44.	b	45.	d	46.	a	47.	b	48.	b	49.	d
50.	a												