HIBA Math Olympiad (HMO)
Sample Paper Grade 10

Pattern and Marking Scheme				
Grade	**Topic / Section**	**NO. of Questions**	**Marks Per Questions**	**Total Marks**
Grade 10	Practical Mathematics	40	1	40
	Achiever's Section	10	2	20
Grade Total		**50**		**60**

The total duration of the exam is 60 minutes. Grade 10 (Age 15–16)

Syllabus

Section 1: Real Numbers, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Arithmetic Progressions, Triangles, Coordinate Geometry, Introduction to Trigonometry, Some Applications of Trigonometry, Circles, Constructions, Areas Related to Circles, Surface Areas and Volumes, Statistics, Probability.

Achievers Section: Higher Order Thinking Questions - Syllabus as per Section 1

Each Question is 1 Mark

1. If $4y + 3x = 2$, $y = x + 6$ and $6y + bx = 5$ are concurrent, find the value of b.

a) 2   b) 4   c) 6   d) 8

2. Two vertices of a triangle are $(4, -2)$ and $(-1, 5)$. If the orthocenter of the triangle is the origin, find the third vertex.

a) (3, 6)   b) (-3, 6)   c) (3, -6)   d) (-3, -6)

3. For an acute angle θ, $\sin\theta + \cos\theta$ takes its maximum value when $\theta =$

a) 25°   b) 35°   c) 45°   d) 55°

4. If the coefficients of the rth and $(r + 1)^{th}$ terms in the expansion of $(1 + x)^{22}$ are in the ratio $2:3$, find the value of r.

a) 9   b) 10   c) 11   d) 12

5. Which term is numerically the greatest term in the expansion of $(5 + 2x)^{30}$, when $x = \frac{1}{4}$?

a) 7th term   b) 8th term   c) 9th term   d) 10th term

6. If the sum of the roots of the equation $ax^2 + bx + c = 0$ equals the product of the roots, then which one of the following is correct?

a) $a^2 + b^2 = c^2$

b) $c(a + b) = ab$

c) $2c + b = 0$

d) $a^2 - b^2 = ac$

7. If $\dfrac{\cos x}{\csc x + 1} + \dfrac{\cos x}{\csc x - 1} = 2$, then which one of the following is one of the values of x?

a) $\dfrac{\pi}{6}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{\pi}{2}$

8. If $p1$ and $p2$ are two odd prime numbers such that $p1 > p2$, then $p1^2 - p2^2$ is:

a) An even number

b) An odd number

c) A prime number

d) A multiple of 4

9. If the first, second, and last terms of an AP are a, b, and c respectively, then the sum is:

a) $\dfrac{(a+c)(c-b)}{(b-a)^2}$

b) $\dfrac{(a+b)(c-a)}{(b-a)^2}$

c) $\dfrac{(b+c)(c-a)}{(b-a)^2}$

d) None of these

10. The areas of two similar triangles are 100 cm² and 64 cm² respectively. What is the ratio of their corresponding medians?

a) 5 : 4   b) 4 : 5   c) 10 : 8   d) 25 : 16

11. To divide a line segment AB in the ratio $3:4$, first a ray AX is drawn such that $\angle BAX$ is an acute angle. Then, equal points are marked on the ray AX. What is the minimum number of points required?

a) 4   b) 5   c) 6   d) 7

12. Write the general term in the expansion of $(x^2 - y)^5$:

a) $(-1)^r \binom{5}{r} x^{10-2r}$

b) $(-1)^r \binom{5}{r} y^{10-2r} x^r$

c) $\binom{5}{r} y^{2r} x^{r-5}$

d) $(-1)^r \binom{5}{r} y^r x^{10-2r}$

13. If α, β are the roots of $x^2 - 4x + 2 = 0$, find the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$

.

a) $2x^2 + 4x + 1 = 0$

b) $4x^2 - 4x + 1 = 0$

c) $2x^2 - 4x + 1 = 0$

d) $4x^2 + 2x - 1 = 0$

14. The houses of a row are numbered consecutively from 1 to 59. If the sum of the numbers of the houses before house number x equals the sum of the numbers of the houses after it, find the value of x.

a) 41   b) 45   c) 49   d) 51

15. Find the values of a and b for which $2x^3 - ax^2 - 50x + b$ is a multiple of $x^2 + 3x - 10$.

a) a=7, b=20

b) a=−7, b=20

c) a=12, b=30

d) a=−12, b=30

16. Find the value of $\dfrac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(c-a)(b-c)(a-b)}$.

a) 1   b) 2   c) 3   d) 0

17. A bag contains 51 cards numbered 1 to 51. Two cards are picked at random without replacement. What is the probability that the sum of the two cards is even?

a) 13/25   b) 26/51   c) 25/51   d) None of these

18. The average weight of A, B, and C is 72 kg. If D joins them, the average becomes 70 kg. If another person E, who is 4 kg heavier than D, replaces A, then the average of B, C, D, and E becomes 71 kg. What is the weight of A?

a) 60 kg   b) 64 kg   c) 68 kg   d) 72 kg

19. The following steps are involved in solving a number problem: a positive number is less than its square by 42. Arrange the steps in correct order:
(A) $x^2 - x = 42$
(B) $x^2 - x - 42 = 0$
(C) $(x - 7)(x + 6) = 0$
(D) $x = 7$

a) ABCD   b) ACBD   c) BACD   d) ABDC

20. A virus doubles itself every 20 minutes. In how much time will it become 256 times its initial value?

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a) 140 minutes   b) 160 minutes   c) 180 minutes   d) 200 minutes

21. If $\dfrac{1}{c+a}, \dfrac{1}{b+c}, \dfrac{1}{a+b}$ are in Arithmetic Progression, then which of the following is true?

a) a^2, b^2, c^2 are in Arithmetic Progression

b) a^2, b^2, c^2 are in Geometric Progression

c) a^2, b^2, c^2 are in Harmonic Progression

d) a^2, c^2, b^2 are in Arithmetic Progression

22. How many 4-digit odd numbers can be formed using the digits 1, 2, 3, 5, 6, and 8 if each digit can be used only once?

a) 240   b) 300   c) 360   d) 420

23. A square is drawn by joining the midpoints of a square of side 20 cm. The process is repeated infinitely. What is the sum of the areas of all such squares?

a) 400 cm²   b) 800 cm²   c) 1200 cm²   d) 1600 cm²

24. The shadow of a tower standing on a level ground is d meters longer when the Sun's altitude is $30°$ than when it is 60°. What is the height of the tower?

a) $d.\dfrac{\sqrt{3}}{2}$ b) $d.\dfrac{\sqrt{3}}{3}$ c) $d.\dfrac{\sqrt{3}}{1-\sqrt{3}}$ d) $d.\dfrac{\sqrt{3}}{1+\sqrt{3}}$

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25. The average score of a class of students is 55. The average score of 20 boys is 52, and the average score of the girls is 61. How many girls are there in the class?

a) 10   b) 12   c) 15   d) 18

26. The arithmetic mean of the cubes of the first nnn natural numbers is:

a) $\dfrac{n^2(n+1)^2}{4n}$

b) $\dfrac{n(n+1)}{2}$

c) $\dfrac{(2n+1)(n+1)}{6}$

d) $\dfrac{n(2n+1)(n+1)}{6}$

27. Which of the following best explains the difference between a solar panel and an electric bulb in terms of energy?

a) A solar panel converts light energy into electrical energy, while a bulb converts electrical energy into light.
b) A solar panel produces heat directly, while a bulb produces current.
c) A solar panel stores electrical energy, while a bulb produces sound.
d) Both store energy in the same way.

28. Solve for $x + y$:

$$\frac{x}{3} + \frac{y}{2} = \frac{7}{12}, \frac{x}{2} + y = 2.$$

a) 111   b) 222   c) 333   d) 444

29. The volume of a pyramid with a square base is $32\ cm^3$. If its height is $4\ cm$, what is the length of each side of the base?

a) 2 cm   b) 3 cm   c) 4 cm   d) 5 cm

30. In how many distinct ways can the letters of the word "BALLET" be arranged?

a) 6!   b) $\dfrac{6!}{2!}$ c) $\dfrac{6!}{3!}$ d) $\dfrac{6!}{2! \times 2!}$

31. 40% of the items in a shop were sold at a profit of 30%, while the remaining were sold at $x\%$ loss. If the overall profit is 5%, find the value of x.

a) 10%   b) 15%   c) 20%   d) 25%

32. Simplify the expression:

$$\frac{(a + b) - (a - b)}{2} - a(a^2 - 3b^2)$$

a) $2b^3$ b) $-2b^3$ c) $a^3 + b^3$ d) $a^3 - b^3$

33. A three-digit number is chosen at random. What is the probability that its digits are consecutive integers in ascending order?

a) $\dfrac{1}{90}$ b) $\dfrac{2}{225}$ c) $\dfrac{4}{225}$ d) $\dfrac{1}{75}$

34. In $\triangle\ XYZ$, $\angle Y = 90°$. Let P, Q, and R be the midpoints of XY, YZ, and XZ, respectively. Which of the following is true?

a) Points X, P, Q, and R are concyclic
b) Points Y, P, Q, and R are concyclic
c) Points Z, Q, P, and R are concyclic
d) All of the above

35. If $\tan A = \dfrac{3}{4}$ and $\tan B = \dfrac{5}{12}$, then which of the following is true?

a) $A + B = \dfrac{\pi}{4}$ b) $A - B = \dfrac{\pi}{4}$

c) $2(A + B) = \dfrac{\pi}{4}$ d) $AB = \dfrac{\pi}{4}$

36. If $\sin A = \dfrac{1}{2}$ and A is an acute angle, then find the value of $\dfrac{\tan A - \cot A}{\csc A + 1}$

a) $-\dfrac{1}{3}$ b) $\dfrac{1}{3}$ c) $-\dfrac{2}{3}$ d) $\dfrac{2}{3}$

37. John and Mike can complete a piece of work in 30 days and 40 days, respectively. If they work on alternate days, in how many days will the work be completed?

a) 34 days b) 34.5 days c) 35 days d) 35.5 days

38. A cylindrical container has a hemispherical raised portion at the bottom with radius 3 cm. If the height of the cylinder (excluding the hemisphere) is 12 cm and its radius is also 3 cm, find the volume of liquid it can hold.
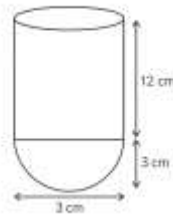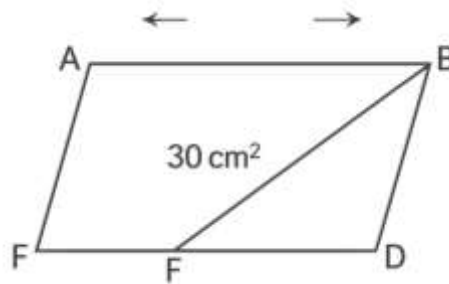

a) $324\pi\ cm^3$ b) $270\pi\ cm^3$ c) $180\pi\ cm^3$ d) $324\pi\ cm^3$

39. In a square PQRS of side $10\ cm$, arcs are drawn with centers at P and R and radius equal to PQ. The overlapping region between the arcs is shaded. Find the area of the shaded region.
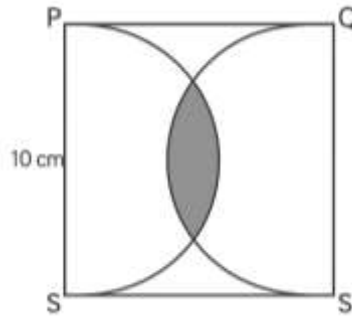
a) 40 cm²   b) 42 cm²   c) 44 cm²   d) 46 cm²

40. In the figure, AB || DE. The area of the parallelogram ABFD is 30 cm². Find the area of triangle AFB.

a) 10 cm²   b) 12 cm²   c) 15 cm²   d) 18 cm²

Each Question is 2 Mark

41. Find the quadratic equation whose roots are the reciprocals of the roots of the equation

$2x^2 - 11x + 7 = 0$

a) $7x^2 - 11x + 2 = 0$

b) $7x^2 + 11x + 2 = 0$

c) $7x^2 - 11x - 2 = 0$

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d) $7x^2 + 11x - 2 = 0$

42. Two regular polygons are such that the ratio of their number of sides is $2:3$, and the ratio of their interior angles is $4:5$. Find the number of sides of each polygon.

a) 6, 9   b) 8, 12   c) 10, 15   d) 12, 18

43. $f(x) = x^4 - 3x^3 + 2x^2 - cx + d$ f(x) = x^4 - is a polynomial such that when divided by $(x - 2)$ and $(x + 2)$, the remainders are 6 and 14 respectively. Find the remainder when $f(x)$ is divided by $(x - 3)$.

a) 8   b) 10   c) 12   d) 14

44. In a rhombus, perpendiculars are drawn from the vertex of the obtuse angles to the opposite sides. The length of each perpendicular is ρ units. If the distance between their feet is q units, then find the area of the rhombus.

a) $\dfrac{pq}{2}$ b) $\sqrt{\rho^2 + q^2}$ c) $\dfrac{\rho^2 + q^2}{2}$ d) pq

45. Inside a triangular field, there is a smaller triangular garden similar to the field. Around the garden, there is a path of uniform width such that the sides of the field are exactly three times the corresponding sides of the garden. Find the ratio of the area of the path to the area of the garden.

a) 8:1   b) 9:1   c) 10:1   d) 7:1

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46. The angles of elevation of the top of a tower from two points at distances ppp and qqq meters from the tower are complementary. If the points and the tower base are collinear, what is the height of the tower?

a) $\sqrt{pq}$ b) p/q   c) q/p   d) pq

47. In the expansion of $(a + b)^n$, $n \geq 6$, the sum of the 6th and 7th terms is zero. Find the $\dfrac{a}{b}$

a) $\dfrac{n-6}{7}$ b) $\dfrac{n-5}{6}$ c) $\dfrac{7}{n-6}$ d) $\dfrac{6}{n-5}$

48. A vertical pole stands on level ground. From a point on the ground 50 m away from the base, the angle of elevation of the top is 45°. Find the height of the pole.

a) 40 m   b) 50 m   c) 60 m   d) 70 m

49. Which of the following sets is a singleton set?

a) The set of even prime numbers
b) The set of odd numbers less than 7
c) The set of perfect cubes less than 30
d) The set of factors of 12

50. A person borrows a sum of money at 12% simple interest per annum and lends half of it at 10% and the other half at 18%. At the end of 10 years, his overall profit is $6,000. How much money did he lend at 18%?

a) $12,000 b) $15,000 c) $18,000 d) $20,000

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Answer Key

1.	b	2.	c	3.	c	4.	b	5.	b	6.	c	7.	b
8.	d	9.	b	10.	a	11.	c	12.	b	13.	a	14.	b
15.	a	16.	b	17.	b	18.	b	19.	a	20.	b	21.	a
22.	b	23.	b	24.	c	25.	b	26.	a	27.	a	28.	b
29.	c	30.	b	31.	b	32.	b	33.	b	34.	d	35.	a
36.	b	37.	b	38.	b	39.	c	40.	a	41.	a	42.	b
43.	c	44.	d	45.	a	46.	a	47.	b	48.	b	49.	a
50.	c												

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